Angular Dependence of the Nuclear Enhancement of Drell-Yan Pairs

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We calculate the nuclear enhancement in the angular distribution of Drell-Yan pairs produced in proton–nucleus reactions. Nuclear effects are encoded in universal twist-4 parton correlation functions. We find that the Lam-Tung relation for the angular coefficients of the lepton-pair distribution holds for the double-hard, but not for the soft-hard contribution. We also predict that nuclear enhancement effects at RHIC energies can be large.

The factorization theorems of QCD [1] permit a rigorous treatment of hard processes by absorbing soft contributions to scattering matrix elements into universal, process-independent distribution functions. The measurement of these distribution functions in one process facilitates the prediction of cross sections for other processes. An example is the Drell-Yan process where two hadrons scatter to produce a lepton pair of high invariant mass Q^2. If Q^2 is large enough, the factorization theorems state that the inclusive Drell-Yan (DY) cross section can be expressed as the product of a calculable hard part and universal twist-2 parton distribution functions that can be measured, for instance, in deep-inelastic lepton-nucleon scattering.

However, the structure of strong interaction dynamics is much richer than can be described solely in terms of twist-2 structure functions, which are two-point correlators and have a probabilistic interpretation in terms of partons. In reactions involving only isolated hadrons, hard processes are usually dominated by a single hard scattering, and correlation functions involving more than two fields are normally suppressed by powers of the large scale. This is different when reactions are considered where individual hadrons interact with nuclei, or one nucleus interacts with another. In that case, a projectile parton can scatter twice within the same nucleus of mass number A, and the double-scattering cross section is enhanced by a factor $A^{1/3}$, compensating for some of the inherent suppression of multi-parton interactions [2,3].

Although we will focus here on proton-nucleus interactions, our work has been primarily motivated by the fact that higher-twist effects should be most prominent in nucleus-nucleus collisions at high energy, as they will be routinely studied at the Relativistic Heavy Ion Collider (RHIC). Many experiments at RHIC will focus on a Q^2 range where higher-twist contributions could be significant [4]. Higher-twist effects will be especially important in the production of minijets, which are thought to provide the main mechanism for the initial energy deposition in the central rapidity region [5,6].

The determination of multi-parton correlation functions in nuclei is also an important step towards the application of QCD to nuclear physics, in general. The enhancement of higher-twist effects in reactions involving nuclei facilitates the extraction of these correlation functions, which encode essential aspects of the difference in the quark-gluon structure between isolated hadrons and nuclei. However nuclear enhancement could point to a general problem, as it might signal an early breakdown of perturbative QCD as applied to high-energy collisions involving nuclei.

In the following we will discuss the Drell-Yan process initiated by a single hadron h scattering off a nuclear target with mass number A:

$$h(P_2) + A(P_1 A) \to l^+ l^- + X \tag{1}$$

At leading-twist level, a parton from the beam hadron reacts with a single parton from the nucleus. Figure 1(a) shows a typical twist-2 diagram describing production of the lepton pair with a high transverse momentum relative to the beam axis. It was already pointed out some time ago by Guo [7] that higher-twist effects in this process are enhanced in this kinematical region (see also [8]). As the parton of the beam hadron travels through the nucleus it can scatter off partons from other nucleons, generating a strong dependence on the size of the nucleus. Such a dependence has been observed in several experiments [9–11] at a scale hard enough to resolve the partonic substructure and thus to allow for a treatment in the framework of perturbative QCD. In Fig. 1(b) we give an example for double scattering.

To extract higher-twist correlation functions from experimental data it is worthwhile to consider observables for which such contributions are large. Ideally, one is looking for an observable with a vanishing contribution from single scattering. Such quantities can be found in the angular distribution of DY pairs. Therefore, we here extend the calculation of Ref. [7] to the differential DY cross section $d\sigma/dQ^2\, dq_\perp^2\, dy\, d\Omega$, keeping the full angular dependence. In addition, we apply our results to $p + A$ collisions at RHIC. Some time ago, Lam and Tung derived a relation [12], similar to the Callan-Gross relation in deep-inelastic scattering, which states that the longitudinal helicity amplitude W_L for the virtual photon in the DY process is exactly twice as large as the double spin-flip amplitude $W_{\Delta\Delta}$. This is a leading-twist prediction valid up to order α_s.

FIG. 1. (a) Schematic single- and (b) double-scattering diagrams for a hadron h colliding with a nucleus A and producing a lepton (l^+l^-) pair. One additional unobserved parton is radiated with high transverse momentum $q_\perp$.

Due to the complex structure of multi-parton correlations a larger number of these correlation functions exist than at leading twist. It is useful to consider observables which receive contributions from only a limited number of correlation functions. We find that the number of contributing correlation functions can be minimized by using different lepton-pair center-of-mass frames (see Ref. [12]). For the DY process, the so-called Gottfried-Jackson (GJ) frame turns out to be the most convenient choice. However, comparing the angular correlations in the GJ and the Collins-Soper (CS) frames also gives valuable insight into the nature of double-scattering contributions [19].

The differential DY cross section is a classical example for a two-scale process in QCD, because both Q^2 and $q_\perp^2$, the transverse momentum of the photon, are detected. In order to avoid the occurrence of large logarithms of the type $\log^2(Q^2/q_\perp^2)$, which are common in this case, we require that both scales are of the same order. This condition also allows us to neglect interference terms that would introduce additional twist-4 matrix elements [2].

The differential cross section for DY production is given by

$$d\sigma = \frac{\alpha^2}{2SQ^4} L_{\mu\nu} W^{\mu\nu} \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{d}\Omega \,, \tag{2}$$

where S is the center-of-mass energy squared, $L_{\mu\nu}$ and $W^{\mu\nu}$ denote the leptonic and hadronic tensors, and the angles θ and ϕ in $\mathrm{d}\Omega$ refer to the polar and the azimuthal decay angles of the lepton pair in the rest frame of the virtual photon. The remaining freedom to choose these angles gives rise to the different choices of, e.g., the GJ and CS frames. The hadronic tensor is given by

$$W_{\mu\nu} = \int \mathrm{d}^4 x \, e^{iq\cdot x} \left\langle P_1 P_2 \right| j_\mu(x) j_\nu(0) \left| P_1 P_2 \right\rangle \ , \tag{3}$$

where P_1 and P_2 are the four-momenta of the nucleus (per nucleon) and the hadron, respectively. The rapidity y and the transverse momentum $q_\perp$ of the lepton pair can be expressed in terms of the hadronic Mandelstam invariants $S = (P_1 + P_2)^2$, $T = (P_1 - q)^2$ and $U = (P_2 - q)^2$ as: $y = \frac{1}{2} \ln((Q^2 - U)/(Q^2 - T))$ and $q_\perp^2 = (Q^2 - U)(Q^2 - T)/S - Q^2$.

It is convenient to express the angular distribution of the lepton pair in terms of the helicity amplitudes $W_{\sigma,\sigma'}$ of the virtual photon. A complete set of such amplitudes is [12]:

$$\begin{aligned} W_{\mathrm{T}} &= W_{1,1}, \quad W_{\mathrm{L}} = W_{0,0}, \quad W_{\Delta\Delta} = W_{1,-1} \\ W_\Delta &= \frac{1}{\sqrt{2}} (W_{1,0} + W_{0,1}) \end{aligned} \tag{4}$$

which are referred to as the transverse, longitudinal, spin-flip, and double spin-flip structure functions. In terms of these helicity structure functions the cross section can be written as

$$\begin{aligned} \frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}q_\perp^2\,\mathrm{d}y\,\mathrm{d}\Omega} &= \frac{\alpha^2}{64\pi^3 S Q^2} \\ & \left(W_{\mathrm{TL}} \left(1 + \cos^2\theta\right) + W_{\mathrm{L}} \left(\frac{1}{2} - \frac{3}{2}\cos^2\theta\right) \right. \\ & \left. + \quad W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta} \sin^2\theta \cos 2\phi \right) \ , \end{aligned} \tag{5}$$

where $W_{\mathrm{TL}} = W_{\mathrm{T}} + \frac{1}{2} W_{\mathrm{L}}$. Integration over the angles reduces the cross section to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}q_\perp^2\,\mathrm{d}y} = \frac{\alpha^2}{12\pi^2 S Q^2} W_{TL} \ . \tag{6}$$

The ratio of (5) and (6) is often parametrized as

$$1 + \lambda\cos^2\theta + \mu \sin 2\theta \, \cos\phi + \frac{\nu}{2} \sin^2\theta \, \cos 2\phi \ . \tag{7}$$

We have verified that we reproduce the known results for the leading-twist matrix elements [14]. In particular, we find $W_{\Delta\Delta} = \frac{1}{2} W_{\mathrm{L}}$, or $\lambda + 2\nu = 1$, which is the Lam-Tung relation.

For the calculation of the double-scattering contributions we use the formalism of Luo, Qiu and Sterman [2,7]. Here we present only our main results and refer the reader to a forthcoming publication for details of the calculation [15]. We factorize the hadronic tensor for all contributions in the usual way, and we keep only those matrix elements, in which the two partons from the nucleus appear separately in combinations of color singlet operators. Assuming the absence of long-range color fluctuations in the nuclear wave function, only such correlators will show a nuclear enhancement.

It is useful to distinguish two contributions to double scattering [2,7]. In the first case (double-hard process), both QCD interactions are hard and the beam parton can be considered as on-shell between the interactions. In the second case (soft-hard process) the beam parton picks up a soft nuclear parton before the hard scattering. The two contributions differ in the pole structure of the hard part of the scattering tensor. We neglect the interference terms between soft and hard rescattering, as it is appropriate when $q_\perp$ is not too small. For $q_\perp^2 \ll Q^2$ the interference terms are important and eventually spoil the nuclear enhancement [7].

First we investigate the case of double-hard scattering. The two-parton distribution functions depend on two Bjorken parameters, $x_a = Q^2/(Q^2 - T)$ and x_h. The cross section is determined by three universal nuclear matrix elements that were introduced in [2]. We only denote the gluon-quark distribution function explicitly:

$$T^{\rm DH}_{qg}(x_a, x_h) = \frac{1}{2x_h} \int \frac{\mathrm{d}z_1^-}{2\pi} \frac{\mathrm{d}z_2^-}{2\pi} \,\mathrm{d}z_3^- \,\Theta(z_1^- - z_2^-) \\ \Theta(-z_3^-) e^{ix_a P_1^+ z_1^-} e^{ix_h P_1^+ (z_2^- - z_3^-)} \\ \langle P_1 |\, F^{\omega +}(z_3^-) F^+{}_\omega(z_2^-) \bar{q}(0) \gamma^+ q(z_1^-) \,| P_1 \rangle \;. \quad (8)$$

Here the superscripts $\pm$ denote the light-cone components of the four-vectors. The phase factors in (8) restrict the integration ranges in such a way that one integral over the longitudinal extension of the nucleus remains. This is the origin of nuclear enhancement. The quark-antiquark and two-quark distribution functions, $T^{\rm DH}_{q\bar{q}}(x_a, x_h)$ and $T^{\rm DH}_{qq}(x_a, x_h)$, are given by similar expressions involving the correlator $\langle \bar{q}\gamma^+ q \bar{q} \gamma^+ q \rangle$. The two-gluon correlator does not contribute to double-hard scattering.

The cross section is given by a convolution of a twist-2 structure function $f_{c/H}(\xi)$ of the hadron, the double hard matrix element and a perturbatively calculable hard part $H^{ab+c}(x_a, x_h, \xi)$:

$$\mathrm{d}\sigma \sim \int \frac{\mathrm{d}\xi}{\xi} f_{c/H}(\xi) T^{DH}_{ab}(x_a, x_h) H^{ab+c}(x_a, x_h, \xi). \quad (9)$$

Here a, b and c denote the involved partons which have to be summed up. Note that $x_h = x_h(\xi)$ depends on ξ.

We find by explicit calculation that the Lam-Tung relation holds for double-hard scattering. In the GJ frame we also find that the double-hard contributions do not generate deviations from the simple $(1 + \cos^2\theta)$ behaviour, i.e. $\mu = \nu = 0$. The explanation for this surprising behavior is that the double-hard process resembles the classical picture of double scattering. For example, a quark from the hadron first scatters on a hard gluon from the nucleus, returns to its mass shell by radiating a gluon with large transverse momentum, and then annihilates on a hard antiquark from the nuclear quark sea.

For soft-hard scattering one finds that the dominant contributions come from two universal two-parton distribution functions, $T^{\rm SH}_{qg}(x)$ and $T^{\rm SH}_{gg}(x)$, which depend on a single Bjorken parameter x. They are obtained as integrals over the nuclear quark-gluon and two-gluon correlators, $\langle F^{\omega+} \bar{q} \gamma^+ q F^+{}_\omega \rangle$ and $\langle F^{\omega+} F^+{}_\omega F^{\lambda+} F^+{}_\lambda \rangle$, respectively. Following Luo et al. [2], we neglect the contributions from two-quark correlators involving one soft quark, because these are far exceeded by the correlators involving a soft gluon. Again, assuming the absence of long-range color fluctuations in the nucleus, an enhancement proportional to the nuclear radius is obtained. Explicit evaluation of the soft-hard cross section reveals that the Lam-Tung relation is violated by this contribution.

Before we present numerical results, we need to specify the nuclear twist-4 matrix elements. In the absence of experimental results, we follow [2,16] and express the two-parton distribution functions $T^{\rm DH}$ as products of the known one-parton distribution functions $f(x)$:

$$T^{\rm DH}_{ab}(x_a, x_h) = \bar{C}(A) A\, f_{a/A}(x_a) f_{b/A}(x_h) \;. \quad (10)$$

Here $f_{a/A}$ is the parton distribution (neglecting shadowing and EMC-type effects) in a nucleus with mass number A. The normalization constant is assumed to grow like the nuclear radius, $\bar{C}(A) = C A^{1/3}$ with $C = 0.072$ GeV^2 [7]. For soft-hard scattering the twist-4 matrix elements only depend on the momentum of the hard parton. We assume that they are proportional to the distribution functions of the hard parton and include the effect of the presence of an additional soft gluon as a renormalization:

$$T^{\rm SH}_{ab}(x) = \lambda^2_{\rm LQSG} A^{4/3} f_{a/A}(x) \quad (11)$$

where $\lambda_{\rm LQSG} = 0.1$ GeV is taken from [17]. We use the twist-2 distribution functions from the CTEQ3M parameterization for the proton [18].

We first discuss the application of our results to the data for 800 GeV protons on W from the experiment E866 at Fermilab [11]. Figure 2 shows the contribution from the single spin-flip amplitude W_Δ to the differential DY cross section (5) in the CS frame. (Note that W_Δ vanishes in $p+p$ collisions for symmetry reasons.) The twist-4 contribution is much larger than the twist-2 result, and it is completely dominated by the double-hard scattering process. Note that the soft-hard contribution is negligible. This feature can be tested by checking experimentally for a violation of the Lam-Tung relation which is exclusively given by soft-hard processes. Furthermore a characteristic property of our soft-hard and double-hard contributions is the specific difference between the CS and GJ frames [15].

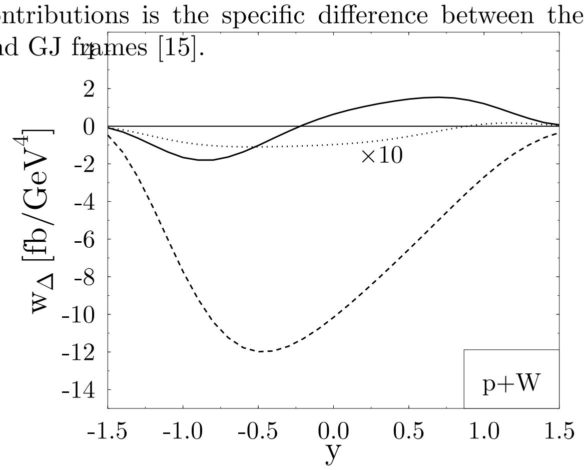

FIG. 2. The helicity amplitude $w_\Delta = \alpha^2 W_\Delta/(64\pi^3 S Q^2)$ for 800 GeV $p + W$ at $Q = 5$ GeV and $q_\perp = 4$ GeV. The plot shows the twist-2 result (solid line), the double-hard (dashed line) and soft-hard (dotted line) twist-4 contributions, in the CS frame. The soft-hard result is multiplied by a factor of 10.

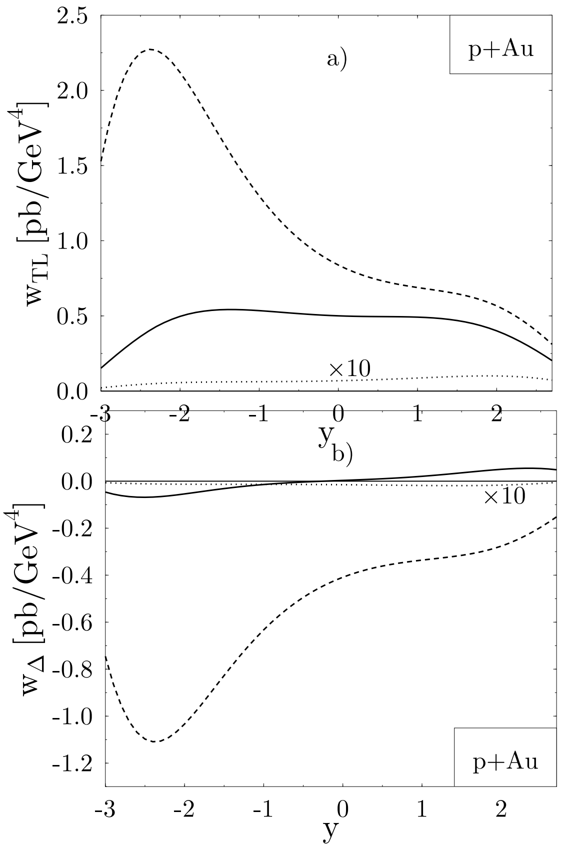

FIG. 3. The helicity amplitudes (a) $w_{\rm TL}$ and (b) w_Δ in the CS frame for 250 GeV protons colliding with 100 GeV/nucleon Au nuclei at $Q = 5$ GeV and $q_\perp = 4$ GeV. For details see Fig. 2.

Next, we turn to $p+Au$ collisions at the full RHIC energy. As Fig. 3 shows, the twist-4 contribution is again larger than the leading-twist result. The twist-4 cross section peaks at negative rapidities (the direction of the proton beam), because the requirement of a high transverse momentum enhances the contribution, in which a quark from the proton annihilates with an antiquark from the Au nucleus. Again, the soft-hard process is negligible. The kinematics of the peak ($Q = 5$ GeV, $q_\perp = 4$ GeV, $y \approx -2$) fits well with the acceptance of the PHENIX detector [20]. The predicted count rate is not high (about 30 events/year in a window of 1 GeV^4), but the characteristic forward-backward asymmetry should be easily detectable.

In summary, we have calculated the nuclear enhancement of the twist-4 contribution to DY production in $p+A$ collisions, with its full angular distribution. For RHIC energies the soft-hard contributions involving highly nontrival multi-field correlators are negligible, which is good news for any comprehensive QCD-description of high-energy heavy ion collisions. The dominant contribution from double-hard scattering has an interpretation in terms of the classical double scattering picture. While double-hard scattering respects the Lam-Tung relation, soft-hard scattering does not. Double-hard scattering does not contribute in the GJ-frame, which allows for a sensitive experimental test of the LQS-formalism. It would be very interesting to calculate the same effect in other approaches [3,21]. A measurement of the nuclear enhanced contribution should be possible at RHIC.

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